Identification and Stochastic Control of an Aircraft Flying in Turbulence

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An adaptive control technique to improve the flying qualities of an aircraft in turbulence was investigated. The approach taken was to obtain maximum likelihood estimates of the unknown coefficients of the aircraft system and then, using these estimates along with the separation principle, to define the stochastic optimal control. The maximum likelihood estimation technique that accounts for the effects of turbulence provided good estimates of the unknown coefficients and of the turbulence. The assessment of the stochastic optimal control based on the maximum likelihood estimates showed that the desired effects were attained for the regulator problem of minimizing pitch angle and the tracking problem of requiring normal acceleration to follow the pilot input.

Nomenclature

A = stability matrix
a_n = normal acceleration, g
a_{n_p} = normal acceleration at the pilot station, g
B = control matrix
C = observation matrix
D = observation control matrix
F = state noise matrix
G = observation noise matrix
$G_{w_g}(\omega)$ = turbulence power spectral density, $(m^2/s^2)/(rad/s)$
g = acceleration due to gravity, m/s^2
H = expected value matrix
H_a = generalized expected value matrix of the state vector
H_n = expected value matrix of the state vector for no feedback control
H_o = expected value matrix of the state vector for the optimal control feedback
J = cost functional
K_A = servoactuator time constant, s
L = row vector defining linear combination states to be minimized
M = pitching moment divided by moment of inertia, rad/s^2
m = output vector due to control input only
n = Gaussian white state noise vector for observations
n_g = Gaussian white state noise, $s^{-1/2}$
P_c = state estimator error covariance matrix for the control formulation
P_s = state estimator error covariance matrix
Q = weighting matrix
R = matrix related to state estimator error covariance matrix
R_L = surface rate limit, deg/s or rad/s
r = variable used to compute optimal control
T = total time interval, s
t = time, s
u = control vector
u_o = optimal control vector
V = mean velocity, m/s
v = observation vector
w_g = vertical turbulence velocity, m/s
X = longitudinal force, N
x = state vector
x_d = state vector due to known control input only
Z = normal force divided by mass and velocity, rad/s
α = angle of attack, deg or rad
α_g = angle of attack induced by vertical velocity component of turbulence, deg or rad
α_s = angle of attack due to vehicle dynamics, deg or rad
γ = desired ratio of a_{n_p} to δ_p, g/rad
δ_e = elevator deflection, deg or rad
δ_p = pilot input to elevator, rad
θ = pitch angle, deg or rad
λ = soft constraint weighting factor
σ = variable of integration
σ_g^2 = turbulence power, m^2/s^2
τ = time variable, s
ω = frequency, rad/s
ω_c = break frequency for turbulence spectrum, rad/s
$\mathcal{L}$ = Volterra operator
Tr = matrix trace function
$\|\cdot\|_{(GG^*)^{-1}}$ = vector norm weighted with respect to $(GG^*)^{-1}$

Superscripts

$\hat{}$ = estimate of superscripted variable
$*$ = matrix transpose

Subscripts

b = bias
c = control form of equations
m = measured
0 = nominal or constant value
$\dot{\alpha},\alpha,\dot{\theta},\delta_e$ = partial derivatives with respect to subscripted variables

Introduction

AIRCRAFT cannot always avoid flying in atmospheric turbulence, so one aircraft design objective is for the aircraft to have satisfactory stability and flying qualities in turbulence. In addition to affecting ride qualities, poor flying qualities in turbulence can be a significant mission problem for vehicles used as stable platforms and for such aircraft as lifting body, space shuttle, and short takeoff and landing

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(STOL) aircraft. Aircraft can be modified aerodynamically to make them less susceptible to turbulence, but these modifications may compromise the overall flying qualities of the aircraft. Another way to improve the flying qualities of the aircraft in turbulence is to define control laws that reduce the aircraft's susceptibility to turbulence.

The definition of these control laws falls within the general area of adaptive control. Many approaches can be taken to the implementation of adaptive control. One such approach is to estimate the unknown stability and control coefficients of the differential equations of motion of the aircraft and then to use these coefficients to determine the stochastic optimal control that minimizes a desired linear combination of aircraft states. The stability and control derivatives and the stochastic optimal control laws can be updated periodically during flight or after the flight is over. The stochastic optimal control is guaranteed by the separation principle,[1,2] which is to obtain estimates of the states from the output of the Kalman filter and then to use these estimates to replace the states in the optimal control laws.

Although there are several well-established techniques for determining aircraft stability and control derivatives from flight data when the tests are performed in smooth air (such as that described in Ref. 3), the methods are generally unable to treat data from flights affected by unknown external disturbances, such as those due to turbulence. The determination of stability and control derivatives from flight-test data affected by significant turbulence disturbances belongs to the class of problems of system identification in the presence of state and observation noise.

This paper presents the results of using the maximum likelihood estimation technique to identify the unknown coefficients of a system in the presence of state and observation noise, specifically the coefficients for the equations for longitudinal short-period aircraft dynamics. The algorithm is used to define the stochastic optimal control laws, and the effect of using these control laws on flight data is demonstrated.

Dynamic System Model for Identification

The system to be identified is the longitudinal short-period dynamics of an aircraft subjected to external turbulence disturbances and specific test inputs by the pilot. The simplified longitudinal equations of motion for this system are as follows:

$$\dot{\alpha}_s = Z_\alpha \alpha_s + \dot{\theta} + Z_{\delta_e}\delta_e + Z_0 + Z_\alpha \alpha_g \tag{1}$$

$$\ddot{\theta} = M_\alpha \alpha_s + M_{\dot{\theta}}\dot{\theta} + M_{\delta_e}\delta_e + M_0 + M_\alpha \alpha_g \tag{2}$$

where

$$\alpha_g = w_g / V \tag{3}$$

and the random vertical turbulence velocity component w_g is considered to have zero mean value and a power spectral density given by the following equation:

$$G_{w_g}(\omega) = \frac{2\sigma_g^2 \omega_c}{\omega^2 + \omega_c^2} \tag{4}$$

where

$$\omega_c = V/300 \tag{5}$$

The above expression is a form of the Dryden turbulence model.[4,5] It should be noted that $M_{\dot{\alpha}}$ and M_q are dependent variables in data obtained in smooth air. However, for data obtained in the presence of state noise (turbulence), Eq. (2) could have included $M_{\dot{\alpha}}$ and $M_{\dot{\alpha}}$ could then be estimated independently of M_q. The relationship of these variables to the aircraft is shown in Fig. 1.

The complete system state and observation equations (including the turbulence model) can be written as:

$$\dot{x} = Ax + Bu + Fn_g \tag{6}$$

$$v = Cx + Du + Gn + v_b \tag{7}$$

where n_g and n are scalar- and vector-valued uncorrelated Gaussian white noise processes with zero means and unity spectral densities, respectively; v_b is the instrument bias vector; and

$$x = \begin{bmatrix} \alpha_s \\ \theta \\ \dot{\theta} \\ \alpha_g \end{bmatrix}, u = \begin{bmatrix} \delta_e \\ \\ 1 \end{bmatrix}, v = \begin{bmatrix} \dot{\theta}_m \\ \theta_m \\ a_{n_m} \\ \alpha_m \end{bmatrix} \tag{8}$$

The unity element in u and the coefficients Z_0 and M_0 in Eqs. (1) and (2) are used to account for possible initial biases in the state equation.

The unknown coefficients to be estimated are contained in matrices A, B, C, D, and F, the vector v_b, and the initial state $x(0)$. Matrices A, B, C, D, F, and G are explicitly defined in Refs. 5-7.

Maximum Likelihood Estimator

The derivation of the maximum likelihood estimation algorithm for the state noise problem is presented in Refs. 1 and 5. It is based on minimizing the cost functional J, which is defined as:

$$J = \frac{1}{T}\int_0^T \Big[\| v - m - \mathcal{L}(v-m) \|^2_{(GG^*)^{-1}} - \| v \|^2_{(GG^*)^{-1}} \Big] dt + Tr(R) \tag{9}$$

where

$$\dot{x}_d = Ax_d + Bu$$

$$m = Cx_d + Du + v_b$$

$$R = (GG^*)^{-1/2} CP_s C^* (GG^*)^{-1/2}$$

and $\mathcal{L}$ is the Volterra operator that is given by the Kalman filter, P_s is the state estimator error covariance matrix, and m

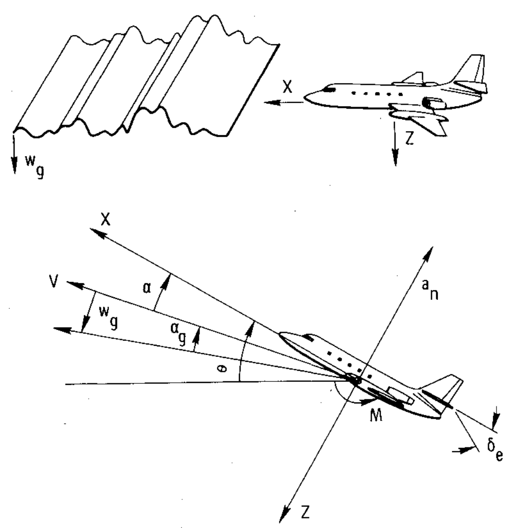

Fig. 1 Axis system and turbulence nomenclature.

is the response of the system due to the control input alone. This algorithm, in contrast to the extended Kalman filter method, uses the Kalman filter to estimate only the states and measurements and not to estimate the unknown coefficients. Thus, this algorithm is quite different from that of the extended Kalman filter. The concept underlying this algorithm is described in more detail in Refs. 5 and 7.

Stochastic Optimal Control

After the unknown coefficients have been identified with one of the maximum likelihood estimators, optimal control theory can be used to minimize any criterion function consisting of linear combinations of the states. Reference 1 shows that, with the separation principle, the stochastic optimal control is defined by standard optimal control theory, except that the estimate of the state $\hat{x}$ replaces the state x in the formulation of the control law. The following discussion summarizes the necessary stochastic optimal control results. A more complete treatment is given in Refs. 1 and 5.

Dynamic Equations for Optimal Control

For convenience, two changes are made to the equations that are used in the identification portion of the problem [Eqs. (6) and (7)]. First, the control is included as one of the states; and second, the aerodynamic bias terms Z_0 and M_0 are eliminated by defining the equations with respect to the trimmed flight condition. The control should be included as an additional state, because the control servoactuator is represented as follows:

$$\dot{\delta}_e(t) = -K_A[\delta_e(t) - \delta_p(t) - u_o(t)] \tag{10}$$

and

$$|\dot{\delta}_e(t)| \leq R_L \tag{11}$$

where K_A is the servoactuator time constant, $\delta_p(t)$ is the known pilot input, $u_o(t)$ is the optimal control input, and R_L is the value of the surface rate limit. The rate limit makes the equations nonlinear, so only optimal control solutions are accepted where the rate limit is not exceeded. Then $\delta_e(t)$ can be defined as an additional state, $x_5(t)$, and Eq. (10), subject to the constraint of Eq. (11), can be written as:

$$\dot{x}_5(t) = K_A[\delta_p(t) + u_o(t) - x_5(t)] \tag{12}$$

where $|\dot{x}_5(t)| \leq R_L$. The following equations can be used to express the state observations:

$$\dot{x}(t) = A_c x(t) + B_c u_o(t) + F_c n_g + B_c \delta_p(t) \tag{13}$$

$$v(t) = C_c x(t) + Gn \tag{14}$$

where the subscript c indicates that the corresponding matrix is for the optimal control phase and not for the identification phase. The matrices A_c, B_c, C_c, F_c, and G are defined in Refs. 5 and 6.

Optimal Control

Two kinds of optimal control formulations are treated herein: the regulator problem, where some linear combination of the states is to be minimized; and the tracking problem, where the resulting optimal control constrains some linear combination of the states to follow a specified parameter. The regulator problem considered here is to minimize the pitch angle excursions $\theta(t)$ in turbulence. The tracking problem considered is to require the normal acceleration at the pilot station to follow the pilot input $\delta_p(t)$ in turbulence. The optimal controls are defined in terms of the estimated state $\hat{x}(t)$, which is the output of the Kalman filter, where the observation vector $v(t)$ is the input to the Kalman filter.

Regulator Problem

The linear combination of the system states to be minimized is defined as $Lx(t)$. Therefore, according to standard optimal control theory the cost functional to be minimized is:

$$J_c = \lim_{T\to\infty} \frac{1}{T}\int_0^T \{[Qx(t), x(t)] + \lambda u_o^2(t)\}\mathrm{d}t \tag{15}$$

where Q equals L^*L, $[Qx(t), x(t)]$ represents the response parameter to be minimized, and $\lambda u_o^2(t)$ is the soft constraint placed on the control to keep the control within the constraint of Eq. (11). For the regulator problem, $\delta_p(t)$ is assumed to be zero. The optimal solution to this equation is:

$$u_o(t) = -B_c^* P_c \hat{x}(t)/\lambda \tag{16}$$

where λ is selected to keep the control within the linear region. The quantity P_c is the solution to the following steady-state Riccati equation:

$$A_c^* P_c + P_c A_c + Q - P_c B_c B_c^* P_c/\lambda = 0 \tag{17}$$

With the results from Refs. 1 and 5, the following equation can be written for $\hat{x}(t)$:

$$\begin{aligned}\dot{\hat{x}}(t) &= [A_c - P_s C_c^*(GG^*)^{-1}C_c]\hat{x}(t)\\ &\quad + P_s C_c^*(GG^*)^{-1}v(t) + B_c u(t)\end{aligned} \tag{18}$$

where P_s, a 5×5 matrix, is the steady-state solution of the following equation:

$$\begin{aligned}\dot{P}_s(t) &= A_c P_s(t) + P_s(t)A_c^* + F_c F_c^*\\ &\quad - P_s(t)C_c^*(GG^*)^{-1}C_c P_s(t)\end{aligned} \tag{19}$$

The theoretical value of the parameter to be minimized can be calculated as:

$$Tr(QH) = \lim_{T\to\infty} \frac{1}{T}\int_0^T [Qx(t), \ x(t)]\mathrm{d}t \tag{20}$$

where H is defined as

$$\begin{aligned}H &= E\{x(t)x(t)^*\} = E\{[\hat{x}(t) - x(t)][\hat{x}(t) - x(t)]^*\}\\ &\quad + E\{x(t)x(t)^*\} = P_s + H_a\end{aligned} \tag{21}$$

and H_a is the solution of the following equation:

$$\begin{aligned}&(A_c - B_c B_c^* P_c/\lambda)H_a + H_a(A_c - B_c B_c^* P_c/\lambda)^*\\ &\quad + P_s C_c^*(GG^*)^{-1}C_c P_s = 0\end{aligned} \tag{22}$$

Thus, the theoretical reduction in the response parameter to be minimized can be calculated, in decibels, as follows:

$$\text{Reduction} = 10\log[Tr(QH_n)] - 10\log[Tr(QH_o)] \tag{23}$$

where H_n is for the open loop response and H_o is for the optimal control response.

Tracking Problem

The tracking problem solution requires that some linear combination of the system states follow an arbitrary parameter. For example, let the normal acceleration at the pilot location $a_{n_p}(t)$ follow the pilot control input $\delta_p(t)$. For the tracking problem the cost functional to be minimized is:

$$J_c = \lim_{T\to\infty} \frac{1}{T}\int_0^T \{[a_{n_p}(t) - \gamma\delta_p(t)]^2 - \lambda u_o(t)^2\}\mathrm{d}t \tag{24}$$

where γ is the desired ratio for $a_{n_p}(t)$ to $\delta_p(t)$, and once again the second term is the soft constraint on the control position. Reference 8 shows the solution to this problem to be

$$u_o(t) = -B_c^* P_c \hat{x}(t)/\lambda + B_c^* r(t)/\lambda \tag{25}$$

where P_c is the same as for the regulator problem. If $r(t)$ is chosen according to theory, future knowledge of $\delta_p(t)$ is required. In practice, if $\delta_p(t)$ is assumed to be a step function, $r(t)$ can be approximated as follows[8]:

$$r(t) = \frac{1}{\tau}\int_{t-\tau}^{t} [(A_c^* - P_c B_c B_C^*)/\lambda]^{-1} L^* \delta_p(\sigma)\,\mathrm{d}\sigma \tag{26}$$

where L is defined by the equation $a_{n_p}(t) = Lx(t)$, and τ is chosen to obtain the desired response to a step input for $\delta_p(t)$. If $\delta_p(t)$ changes slowly, τ probably can be chosen near zero.

Results

Maximum Likelihood Estimates

The usefulness of any algorithm depends on its ability to produce meaningful results from experimental data. The following results were obtained by applying the algorithms to flight data. Approximately 65 s of data (Fig. 2) were obtained from a JetStar[9] aircraft flying in turbulence. The data were acquired during an interval in which turbulence was continuous and the pilot made five prominent inputs. Five maneuvers of approximately equal length (13 s), referred to here as maneuvers A, B, C, D, and E, resulted from these inputs.

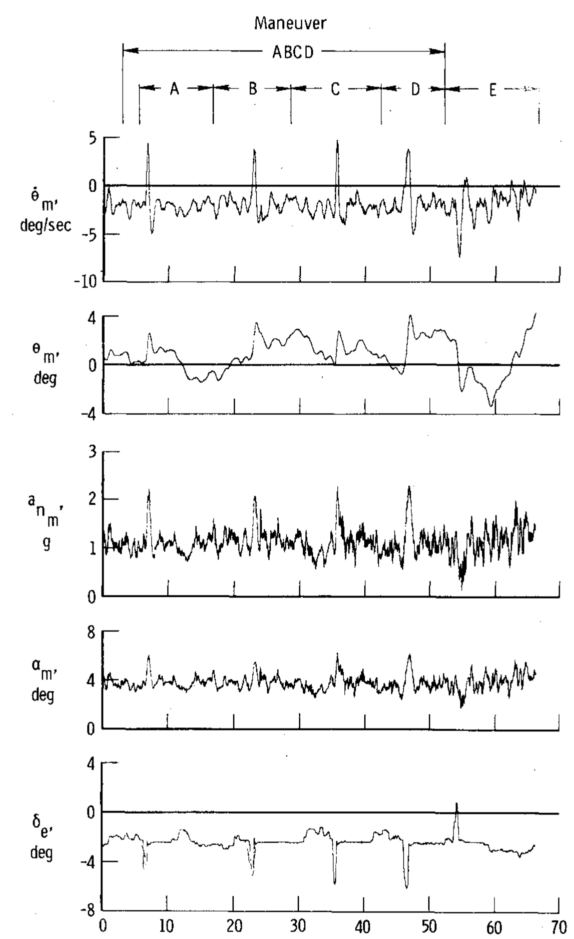

Fig. 2 Total aircraft turbulence time history showing time interval for each manuever.

The need to account for turbulence or state noise when the maximum likelihood estimation algorithm is applied becomes obvious when the results obtained when the state noise is assumed to be negligible are examined.[5-7] Figure 3 shows the comparison when the maximum likelihood estimation algorithm that accounts for the turbulence is applied. The agreement of these two sets of time histories is excellent.

Each of the five maneuvers was analyzed,[5] and each provided acceptable matches between the computed and flight data. The mean values and the standard deviations of the estimates of the stability and control derivatives obtained from these five maneuvers are given in Refs. 5-7, along with flight-determined estimates for smooth air and wind tunnel

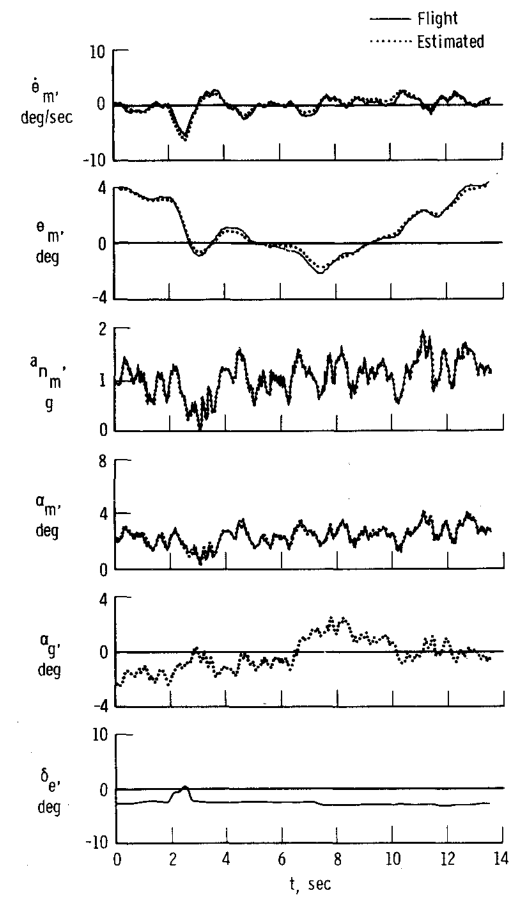

Fig. 3 Comparison of flight data for maneuver E with values estimated by using the algorithm.

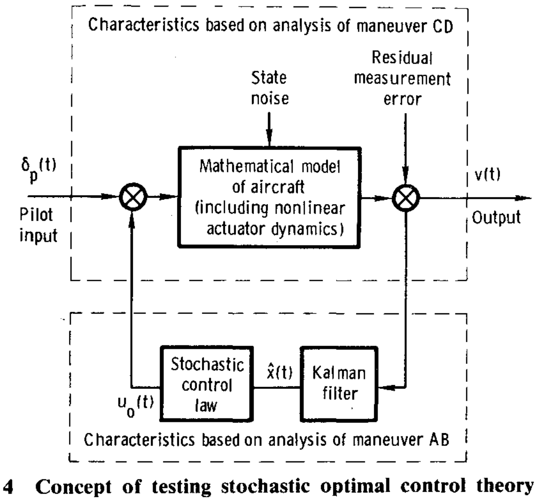

Fig. 4 Concept of testing stochastic optimal control theory using flight data.

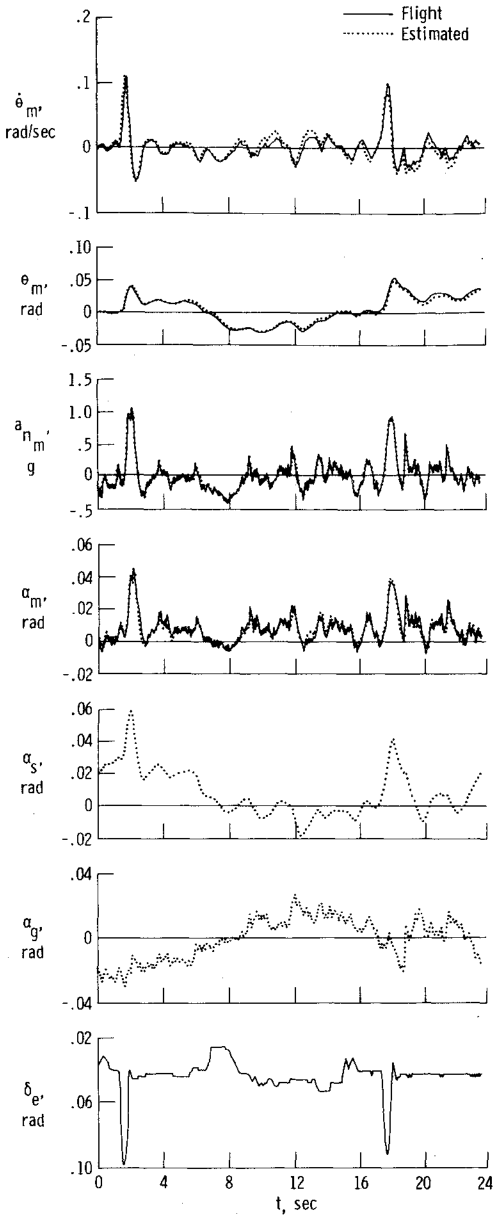

Fig. 5 Comparison of flight data from maneuver AB with estimated values.

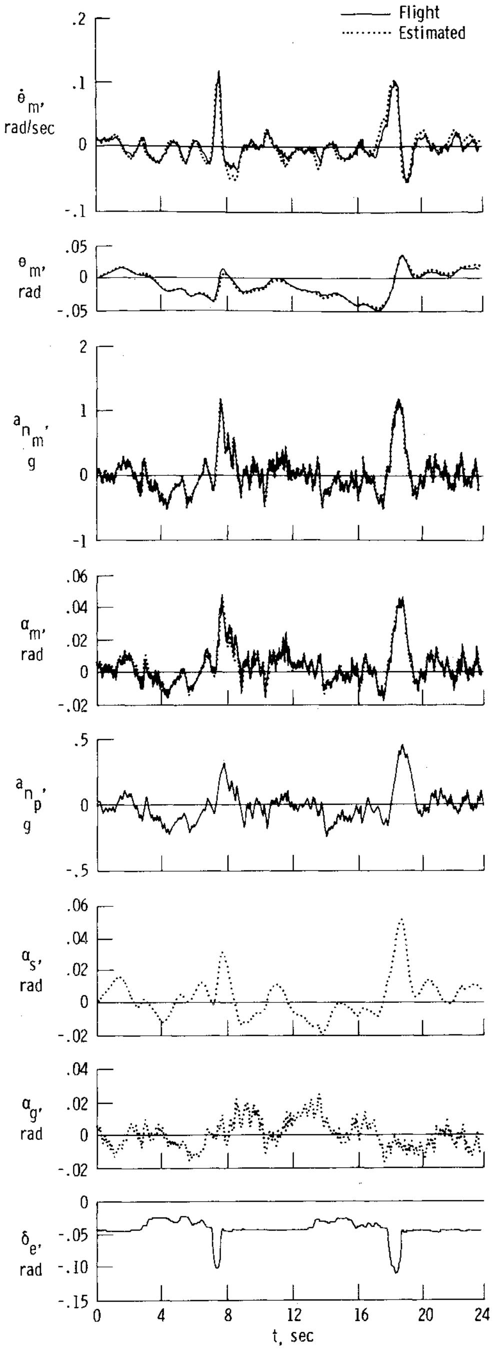

Fig. 6 Comparison of flight data from maneuver CD with estimated values.

estimates. The estimates of the stability and control derivatives obtained were found to be in acceptable agreement with derivatives estimated from flight data obtained in smooth air and with wind tunnel predictions.

To complete the evaluation of the algorithm, its estimates of the state noise were compared with those defined by the Dryden model. As pointed out previously, the power spectral density of atmospheric turbulence can be approximated by the Dryden expression. References 5 and 6 show the comparison of the power spectral density of the estimated turbulence for maneuver ABCD (Fig. 2) with the asymptote for the Dryden expression. The level of the Dryden asymptote is based on the mean square of the estimated turbulence power. The shape of the power spectrum shows excellent agreement with the asymptote.

Thus, the algorithm was found to provide good estimates of the unknown coefficients and the state noise and to yield estimates in good agreement with the flight-test responses. (The results of a more extensive analysis are reported in Refs. 7 and 10, and all of these results are reported in Ref. 5.) Therefore, the stochastic optimal control was investigated by using the maximum likelihood estimates that resulted from the algorithm.

Stochastic Optimal Control Results

Since flight tests have not been made with the previously defined optimal control laws implemented, the effects of these laws were assessed in another way. The approach used was to observe the effect on a data set like maneuver CD (maneuver C plus maneuver D in Fig. 2) of a control law based on an independent set of data, like maneuver AB (maneuver A plus maneuver B). The process is shown in the block diagram in Fig. 4. The elements of the forward loop are the mathematical model of the aircraft, the estimated state noise, and the residual measurement error determined from the analysis of maneuver CD. The elements of the feedback loop, that is, the stochastic control law and the Kalman filter, are defined by the analysis of maneuver AB. The Kalman filter was used to obtain $\hat{x}(t)$ and the stochastic control law was used to obtain $u_o(t)$. The diagram gives an approximation of what would occur if the control law were implemented in an aircraft (i.e., the Kalman filter and the control law must be based on a prior

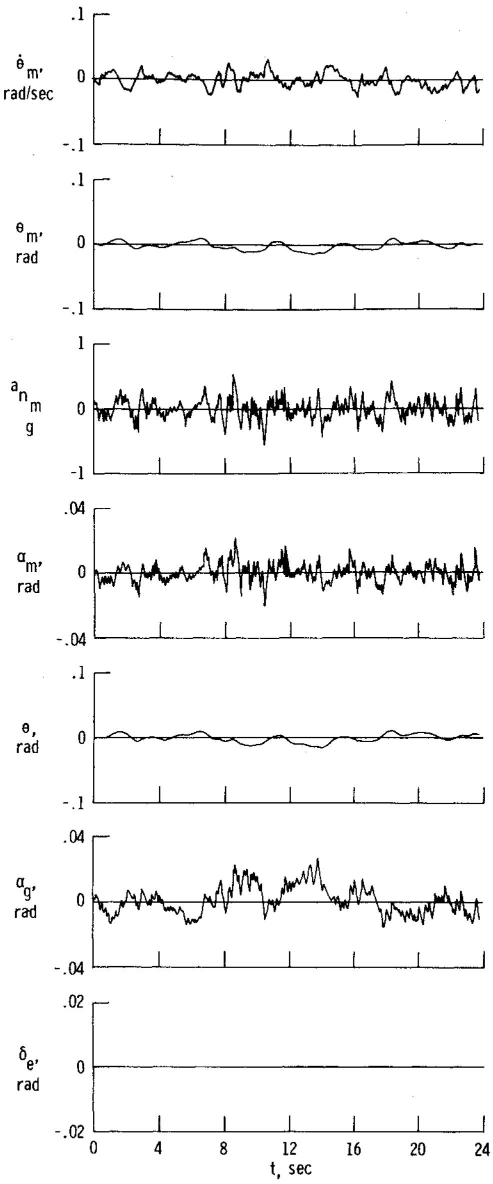

Fig. 7 Calculated noise-contaminated response to turbulence only for time interval corresponding to maneuver CD.

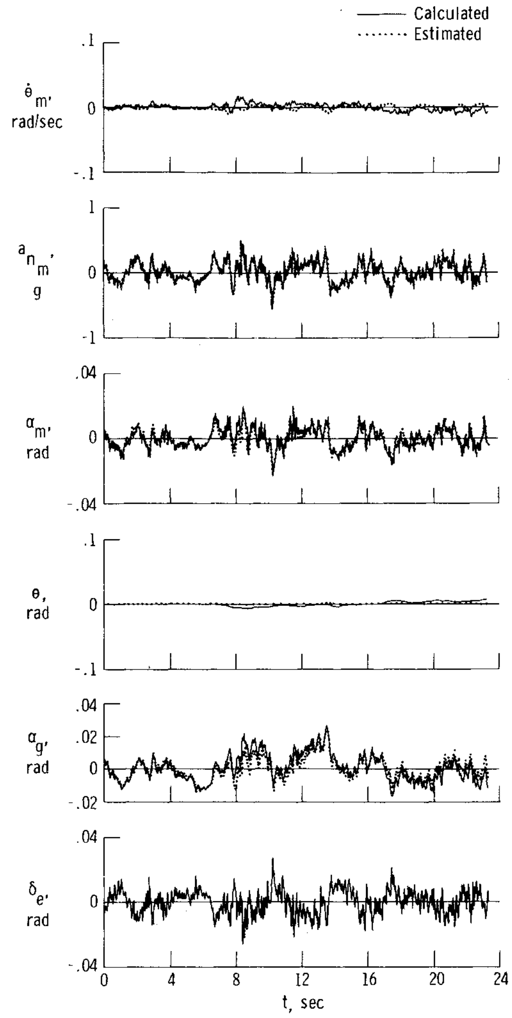

Fig. 8 Comparison of calculated and estimated optimal control to minimize θ^2 by using technique in Fig. 4.

data set). Figure 5 shows that the match of the estimates provided by the algorithm with the flight data for maneuver AB is good, as is the match of the estimates and flight data for maneuver CD (Fig. 6).

Although both matches are good, there is, however, significant residual error in the estimates for maneuver CD when control inputs are large. This means that the residual measurement error to be added to the output of the mathematical model of the aircraft (Fig. 4) is probably larger and more highly correlated than it would be during the implementation of a regulator problem on an aircraft in flight, because the control inputs are smaller. If the feedback loop shown in Fig. 4 is eliminated and $\delta_p(t)$ is the same as the pilot input for maneuver CD, the system output $v(t)$ becomes identical to the measurement of maneuver CD.

Regulator Problem

If the pilot input is set identically equal to zero and the feedback loop is eliminated, the resulting output is only the aircraft response to turbulence during the time interval for maneuver CD. The resulting output is shown in Fig. 7. There are significant variations in all the output responses, but the one of most interest here is pitch angle $\theta(t)$. The regulator solution for the minimization of the variation in $\theta(t)$ [Eq. (15)] can then be tested as it would be in flight, by closing the feedback loop and allowing the stochastic optimal control $u_o(t)$ from Eqs. (16-19) (which are based on the estimates from maneuver AB) to minimize $\theta(t)$. The result of a test of this type is shown in Fig. 8, where the state $\theta(t)$ is very small and the estimate of $\theta(t)$, that is, $\hat{\theta}(t)$, is nearly zero for the entire time interval. It should be noted that the residual measurement error for the estimates of $\theta(t)$ is larger at the times corresponding to the larger pilot control inputs for the original time history of maneuver CD. This shows that the test of the control law is probably more stringent than it would be if the control law were actually implemented on an aircraft. The theoretical reduction of $\theta(t)$ based on Eq. (23) for these data is 95%. The reduction in θ for the data shown in Figs. 7 and 8 can also be calculated. The calculation of the mean square of $\theta(t)$ for no feedback yields 0.58×10^{-4} rad^2, and the calculation of the mean square for $\theta(t)$ with the stochastic optimal control is 0.91×10^{-5} rad^2, a reduction of 84%. This comparison shows the agreement between the actual and theoretical reductions in $\theta(t)$ to be good. The results in Fig. 8 show that the solution to the regulator problem developed in this paper can successfully be applied to the minimization of pitch angle for an aircraft flying in turbulence. More detailed results from these data and simulated data are contained in Ref. 5.

Tracking Problem

The tracking problem can be investigated in the same way as the regulator problem – by using the technique shown in Fig. 4. The differences are that the parameter considered is

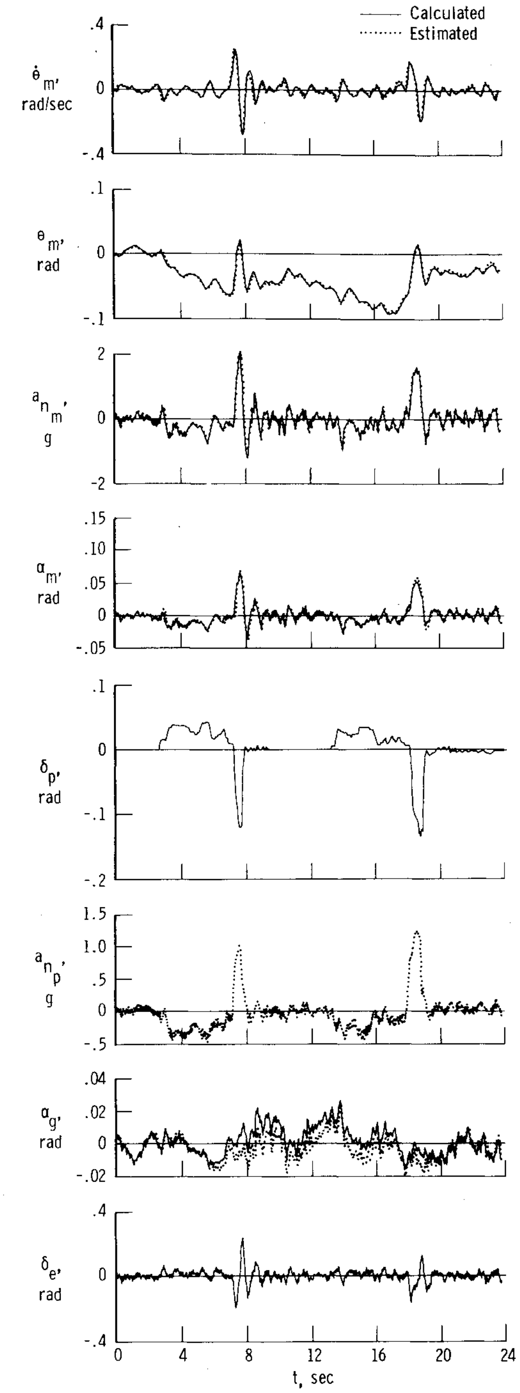

Fig. 9 Comparison of calculated and estimated optimal control to require a_{n_p} to follow $-20\delta_p$ by using technique in Fig. 4.

$\delta_p(t)$ from maneuver CD, and that the control law used results from the minimization of the cost functional in Eq. (24). A value of γ equal to -20 g/rad was chosen as the desired ratio for the response of $a_{n_p}(t)$ to the pilot input $\delta_p(t)$. This means that the response $a_{n_p}(t)$ should track -20 times the pilot input $\delta_p(t)$. The value of τ [Eq. (26)] was set at 0.02 s. The results of implementing the tracking stochastic optimal control defined by Eqs. (25) and (26) are shown in Fig. 9. All the matches are satisfactory, and the general shape of $a_{n_p}(t)$ is in good agreement with the tracked parameter $\delta_p(t)$. This comparison can be seen more clearly in Fig. 10, where $-20\delta_p(t)$ is superimposed on $a_{n_p}(t)$. Figure 10 shows that the tracking problem solution results in excellent agreement between the tracked variable and the tracking variable. Figure 11 shows $a_{n_p}(t)$ from the original maneuver CD, with $-20\delta_p(t)$ from maneuver CD superimposed on it.

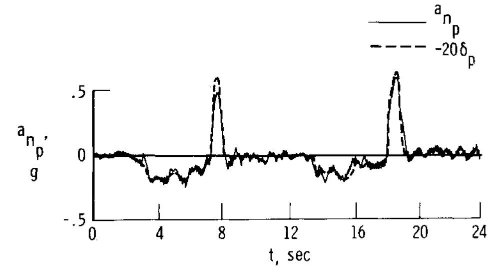

Fig. 10 Comparison of $-20\delta_p$ from maneuver CD with a_{n_p} from Fig. 9 with stochastic optimal control implemented.

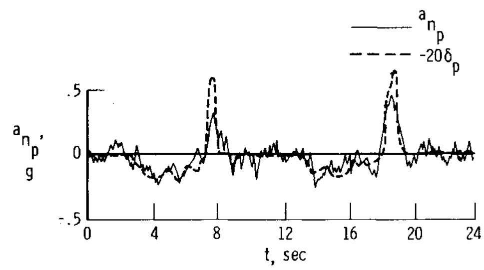

Fig. 11 Comparison of $-20\delta_p$ from maneuver CD with a_{n_p} from maneuver CD without stochastic optimal control implemented.

The comparison is much worse than the comparison in Fig. 10, largely because the turbulence component in Fig. 10 is much smaller than in Fig. 11. This comparison shows that the tracking problem solution developed in this study can successfully be applied to an aircraft flying in turbulence.

The overall technique discussed shows promise for improving aircraft flying qualities in turbulence. Further flight tests should be made to validate the technique completely.

Concluding Remarks

In order to improve the flying qualities of an aircraft in turbulence, an adaptive control technique was investigated. This approach involved obtaining maximum likelihood estimates of the unknown coefficients of the aircraft system, and then, using these estimates along with the separation principle, defining the stochastic optimal control for the regulator and tracking problem of an aircraft flying in turbulence.

The maximum likelihood estimates obtained from an estimator that accounted for the turbulence (state noise) were found to: 1) provide computed responses in good agreement with measured aircraft flight responses; 2) result in estimates of coefficients that were in good agreement with flight-determined estimates obtained in the absence of turbulence and with wind tunnel estimates; and 3) provide estimates of turbulence in good agreement with the assumed Dryden model. An assessment of the stochastic optimal control based on the maximum likelihood estimates showed that the desired effects were attained for the regulator problem of minimizing pitch angle and the tracking problem of requiring normal acceleration to follow pilot input. The results of the regulator problem were in good agreement with the theoretical predictions. The technique shows promise for improving aircraft flying qualities in turbulence, and more flight tests should be undertaken to further validate it.

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